UNIVERSITY OF MUMBAI

Program: M.A. /M.Sc.

Course: Mathematics

Syllabus for: Semester I and Semester II

(Modified)

(Credit Based Semester and Grading System
with effect from the Academic Year 2013–2014)

M. A. /M.Sc. Mathematics Syllabus for Semester I and Semester II
Credit Based Semester and Grading System
To be implemented from the Academic Year 2013-2014

SEMESTER I

Algebra I				
Course Code	UNIT	TOPICS	Credits	L/ Week
PSMT101 PAMT101	I	Linear Transformations	5	4
	II	Determinants		
	III	Characteristic polynomial		
	IV	Inner product spaces, Bilinear forms		
Analysis I				
PSMT102 PAMT102	I	Metric Spaces, Euclidean space IR^n	5	4
	II	Continuous functions		
	III	Differentiable functions		
	IV	Inverse function theorem, Implicit function theorem.		
Complex Analysis I				
PSMT103 PAMT103	I	Power series	5	4
	II	Complex differentiability		
	III	Analytic Functions		
	IV	Cauchy's theorem		
Discrete Mathematics				
PSMT104 PAMT104	I	Number theory	5	4
	II	Advanced counting		
	III	Pigeon-hole principle		
	IV	Boolean algebra, Graph theory		
Set Theory and Logic				
PSMT105 PAMT105	I	Introduction to logic	4	4
	II	Sets and functions		
	III	Partial order		
	IV	Permutations		

SEMESTER II

Course Code	UNIT	TOPICS	Credits	L/ Week
Algebra II				
PSMT201 PAMT201	I	Groups, group Homomorphisms	5	4
	II	Groups acting on sets		
	III	Rings		
	IV	Divisibility in integral domains		
Topology				
PSMT202 PAMT202	I	Topological spaces	5	4
	II	Connected topological spaces		
	III	Compact topological spaces		
	IV	Compact metric spaces, Complete metric spaces		
Complex Analysis II				
PSMT203 PAMT203	I	Cauchy's theorem	5	4
	II	Maximum Modulus and Open Mapping Theorem		
	III	Singularities		
	IV	Residue calculus		
Differential Equations				
PSMT204 PAMT204	I	Picard's Theorem	5	4
	II	Ordinary Differential Equations		
	III	Sturm- Liouville Theory		
	IV	First Order Nonlinear Partial Differential Equation		
Elementary Probability Theory				
PSMT205 PAMT205	I	Probability basics	4	4
	II	Probability measure		
	III	Random variables		
	IV	Limit Theorems		

Teaching Pattern:

1. Four lectures per week per course (1 lecture/period is of 60 minutes duration).
2. In addition, there will be tutorials, seminars, as necessary for each of the five courses.

M. Sc. Mathematics Syllabus
Credit and Grading System
To be implemented from the Academic year 2013-2014

Semester I

PSMT101/PAMT101: Algebra I

Unit I. Linear Transformations (15 Lectures)

Linear equations, Vector spaces, Review of Linear Equations, Homogeneous and non-homogeneous system of linear equations, Gaussian Elimination, Vector spaces, linear independence, basis, dimension, dual spaces, dual basis, the double dual, Linear transformations, kernel and image, rank and nullity of a linear transformation, relationship with linear equations, the algebra of linear transformations, relationship with matrices, rank of a matrix, transpose of a linear transformation, invertible operators.

Unit II. Determinants (15 Lectures)

Determinants as alternating n-forms, existence and uniqueness, Laplace expansion of determinant, determinants of products and transpose, determinants and invertible operators, determinant of a linear operator.

Unit III. Characteristic polynomial (15 Lectures)

Eigenvalues and Eigenvectors of a linear transformation, Characteristic polynomial, Minimal polynomial, Cayley Hamilton Theorem, Triangulable and diagonalizable linear operators, invariant subspaces, nilpotent operators, Jordan Canonical Form (Statement only).

Unit IV. Inner product spaces, Bilinear forms (15 Lectures)

Inner product spaces, orthonormal basis, Adjoint of an operator, Unitary operators, Self adjoint and normal operators, Spectral theorems, Bilinear forms, Symmetric bilinear forms, Classification.

Reference Books:

1. Hoffman K, Kunze R : Linear Algebra, Prentice-Hall India.
2. Serge Lang: Linear Algebra, Springer-Verlag Undergraduate Text in Mathematics.

Recommended Books:

1 Michael Artin: Algebra, Prentice-Hall India.

PSMT102/PAMT102: Analysis I

Unit I. Metric Spaces, Euclidean space R^n (15Lectures)

Metric spaces, examples, examples of function spaces, open and closed sets, equivalent metrics, sequences and convergence in metric spaces, metrics on R^n equivalent metrics on R^n, Standard topology on R^n, Compact subsets of R^n, Connected subsets of R^n, Bolzano-Weierstrass Theorem, Heine-Borel Theorem, Lebesgue Covering Lemma.

Unit II. Continuous functions (15 Lectures)

Continuous functions on R^n, sequential continuity, continuity and compactness, Uniform continuity, continuity and connectedness.

Unit III. Differentiable functions (15 Lectures)

Differentiable functions on R^n, Total derivative, Partial derivatives, directional derivatives, Chain rule, derivatives of higher order, C^k- functions, $C^\infty-$ functions.

Unit IV. Inverse function theorem, Implicit function theorem (15 Lectures)

Mean value theorem, maxima, minima, Taylor expansion, Inverse function theorem, Implicit function theorem. (Inverse implies Implicit).

Reference Books:

1. Walter Rudin: Principles of Mathematical Analysis, Mcgraw-Hill India.
2. Tom Apostol: Mathematical Analysis, Narosa.

Recommended Books

1. Richard R. Goldberg: Methods of Real Analysis, Oxford and IBH Publishing Company.
2. Michael Spivak: Calculus on Manifolds, Harper-Collins Publishers.
3. Steven Krantz: Real analysis and foundations, Chapman and Hall.
4. Charles Chapman Pugh: Real mathematical analysis, Springer UTM

PSMT103/PAMT103: Complex Analysis I

Unit I. Power series (15 Lectures)

Complex Numbers, Geometry of the complex plane, Riemann sphere, Complex sequences and series, Ratio and root tests for convergence, Sequences and series of functions in C, Uniform convergence, Weierstrass's M-test, Complex power series, radius of convergence of a power series.

Unit II. Complex differentiability (15 Lectures)

Complex differentiable functions, Holomorphic functions, Cauchy Riemann equations, Mobius transformations.

Unit 1II. Analytic functions (15 Lectures)

Analytic functions, Analytic functions are holomorphic, Trigonometric functions, Exponential function, Branches of logarithm.

Unit IV. Cauchy's theorem (15 Lectures)

Complex functions and Line integrals, primitive, Cauchy-Goursat Theorem for a triangle.

Reference Books:

1. Walter Rudin, Principles of Mathematical Analysis, McGraw-Hill, India (For Unit I).
2. Serge Lang: Complex Analysis, Springer.
3 . James W. Brown & R. V. Churchill: Complex variables and applications, Mcgraw-Hill Asia.

Recommended Books:

1. J. B .Conway: Functions of one complex variable, Narosa.
2. A. R. Shastri: An introduction to complex analysis, Macmillan.
3. R. Remmert: Theory of complex functions, Springer.

PSMT104/PAMT104: Discrete Mathematics

Unit I. Number theory (15 Lectures)

Divisibility, Linear Diophantine equations, Cardano's Method, Congruences, Arithmetic functions τ, φ, σ. The sum rule and the product rule, two-way counting, permutations and combinations, Binomial and multinomial coefficients, Pascal identity, Binomial and multinomial theorems.

Unit II. Advanced counting (15 Lectures)

Advanced counting: Types of occupancy problems, distribution of distinguishable and indistinguishable objects into distinguishable and indistinguishable boxes (with condition on distribution) Stirling numbers of second and first kind, Inclusion Exclusion Principle and its application to derangement.

Unit III. Pigeon-hole principle (15 Lectures)

Mobius inversion formula on a partially ordered set, Pigeon-hole principle, generalized pigeon-hole principle and its applications, Erdos- Szekers theorem on monotone subsequences,

Unit IV. Boolean algebra, Graph theory (15 Lectures)

Lattices, Distributive and Modular Lattices, complements, Boolean Algebra, Boolean expressions, Applications, Elementary Graph theory, Hand-shaking Lemma, Paths, Walks, Trails, Cycles, Connected Graphs, Trees.

Reference Books:

For Unit I: 1. Burton, Introduction to Number Theory

2. Nadkarni and Telang: Introduction to Number Theory

For Unit II and III: Richard A. Brualdi: Introductory Combinatorics, Pearson

For Unit IV: J. A. Bondy and U. S. R. Murty: Graph Theory, Springer Verlag.

2 . Kenneth Rosen: Discrete Mathematics and its applications, Tata McGraw Hills.

Recommended Books

1. V. Krishnamurthy: Combinatorics: Theory and applications, Affiliated East-West Press.

2. A. Tucker: Applied Combinatorics, John Wiley & Sons.

3. Norman L. Biggs: Discrete Mathematics, Oxford University Press.

PSMT105/ PAMT105: Set Theory and Logic

Unit I. Introduction to logic (15 Lectures)

Statements, Propositions and Theorems, Truth value, Logical connectives and Truth tables, Conditional statements, Logical inferences, Methods of proof, examples, Basic Set theory: Union , intersection and complement, indexed sets, the algebra of sets, power set, Cartesian product, relations, equivalence relations, partitions, discussion of the example congruence modulo-m relation on the set of integers.

Unit II. Sets and functions (15 Lectures)

Functions, composition of functions, surjections, injections, bijections, inverse functions, Cardinality Finite and infinite sets, Comparing sets, Cardinality, $|A| < |P(A)|$, Schroeder-Bernstein theorem (With Proof) , Countable sets, Uncountable sets, Cardinalities of N , $N \times N$, Q, R, $R \times R$.

Unit III. Partial order (15 Lectures)

Order relations, order types, partial order, Total order, Well ordered sets, Principle of Mathematical Induction, Russell's paradox, Statements of the Axiom of Choice, the Well Ordering Principle, Zorn's lemma, applications of Zorn's lemma to maximal ideals and to bases of vector spaces.

Unit IV. Permutations (15 Lectures)

Permutations, decomposition into cycles, product of permutations, permutations and geometric symmetry, computing the order of a permutation, even and odd permutations.

Reference Books:

For Unit I: Robert R. Stoll: Set theory and logic, Freeman & Co.

For Unit II: 1. James Munkres: Topology, Prentice-Hall India;

2. J. F. Simmons, Introduction to Topology and real analysis.

For Unit III: Joseph A. Gallian: Contemporary Abstract Algebra, Narosa.

Recommended Books

1. Larry J. Gerstein: Introduction to mathematical structures and proofs, Springer.
2. Joel L. Mott, Abraham Kandel, Theodore P. Baker: Discrete mathematics for Computer scientists and mathematicians, Prentice-Hall India.
3. Robert Wolf: Proof, logic and conjecture, the mathematician's toolbox, W. H. Freemon.

Semester II

PSMT201/ PAMT201: Algebra II

Unit I. Groups, group Homomorphisms (15Lectures)

Groups, examples, subgroups, cyclic groups, Lagrange's theorem, normal subgroups, Examples such as Permutation groups, Dihedral groups, Matrix groups, Groups of units in Z_n, Group homomorphisms, Quotient groups, direct product of groups, homomorphisms, isomorphism theorems, automorphisms, inner automorphisms, Structure theorem of finite Abelian groups (statement only) and applications.

Unit II. Groups acting on sets (15Lectures)

Center of a group, Centralizer, Normalizer, Groups acting on sets (general case), Class equation, p-groups, Sylow theorems, Classification of groups upto order 15.

Unit III. Rings (With Unity) (15 Lectures)

Rings, ideals, integral domains and fields, ring homomorphisms, quotient rings, Isomorphism theorems, polynomial rings, Quotient field, Characteristic of a ring.

Unit IV. Divisibility in integral domains (15 Lectures)

Prime elements, irreducible elements, Unique Factorization Domains, Principal Ideal Domains, Gauss's lemma, Z[X] is a UFD, Irreducibility criteria, Eisenstein's criterion, Euclidean domains.

Reference Books:

1. Joseph A. Gallian: Contemporary Abstract Algebra, Narosa.
2. N. S. Gopalkrishnan, University Algebra.

Recommended Books

1. Michael Artin: Algebra, Prentice-Hall India.
2. I. N. Herstein: Topics in Algebra, Wiley-India.
3. David Dummit, Richard Foot: Abstract Algebra, Wiley-India.
4. J. B. Fraleigh, A First Course in Abstract Algebra.

PSMT202/PAMT202: Topology

Unit I. Topological spaces (15 Lectures)

Topological spaces, basis, subbasis, product topology, subspace topology, closure, interior, continuous functions, Quotient spaces, T_1, T_2 spaces.

Unit II. Connected topological spaces (15 Lectures)

Connected topological spaces, Path-connected topological spaces, continuity and connectedness, local connectedness, Connected components of a topological space, Path components of a topological space, Countability Axioms, Separation Axioms, Separable, Lindeloff and Second countable.

Unit III. Compact topological spaces (15 Lectures)

Compact spaces, limit point compact spaces, continuity and compactness, Tube lemma, compactness and product topology, local compactness, one point compactification, Regular topological spaces, Normal topological spaces.

Unit IV: Compact metric spaces, Complete metric spaces (15 Lectures)

Complete metric spaces, Completion of a metric space, Total boundedness, compactness in Metric spaces, sequentially compact metric spaces, uniform continuity, Lebesgue covering lemma.

Reference Books:

1. James Munkres: Topology, Pearson.

Recommended Books

1. George Simmons: Topology and Modern Analysis, Tata Mcgraw-Hill.
2. M. A. Armstrong: Basic Topology, Springer UTM.

PSMT203/ PAMT 203: Complex Analysis II

Unit I. Cauchy's theorem (15 Lectures)

Cauchy's theorem (homotopy version) and applications, Winding number, Cauchy integral formula, Cauchy's estimate, Power series expansion for a holomorphic function, Entire functions, Liouville's theorem, Morera's theorem.

Unit II Maximum Modulus and Open Mapping Theorem (15 Lectures)

Holomorphic functions and their properties: Maximum modulus theorem, Open mapping theorem, zeros of analytic functions, Identity theorem, Schwartz's lemma, Automorphisms of a disc.

Unit III. Singularities (15 Lectures)

Isolated singularities, removable singularities, poles and essential singularities, Laurent Series, Riemann's removable singularity theorem, Casorati-Weierstrass theorem.

Unit IV. Residue calculus (15 Lectures)

Residue Theorem and its applications, evaluation of standard types of integrals by the residue calculus method, Argument principle, Rouche's theorem.

Reference Books:

1. J. B. Conway: Functions of one complex variable, Narosa.
2. Serge Lang: Complex Analysis, Springer.

Recommended Books

1. James W. Brown & R. V. Churchill: Complex variables and applications, Mcgraw-Hill Asia.
2. A. R. Shastri: An introduction to complex analysis, Macmillan.
3. R. Remmert: Theory of complex functions, Springer.
4. Priestley, H. A.: Introduction to Complex Analysis.

PSMT204 / PAMT 204: Differential Equations

Unit I. Picard's Theorem (15 Lectures)

Existence and Uniqueness of solutions to initial value problem of first order ODE- both autonomous, non-autonomous (Picard's Theorem), Picard's scheme of successive Approximations, system of first order linear ODE with constant coefficients and variable coefficients, reduction of an n-th order linear ODE to a system of first order ODE.

Unit II. Ordinary Differential Equations (15 Lectures)

Existence and uniqueness results for an n-th order linear ODE with constant coefficients and variable coefficients , linear dependence and independence of solutions of a homogeneous n-th order linear ODE, Wronskian matrix, Lagrange's Method (variation of parameters), algebraic properties of the space of solutions of a non-homogeneous n-th order linear ODE.

Unit III. First order partial differential equations (15Lectures)

Solutions in the form of power series for second order linear equations of Legendre and Bessel, Legendre polynomials, Bessel functions. Sturm- Liouville Theory: Sturm-Liouville Separation and comparison Theorems, Oscillation properties of solutions, Eigenvalues and eigenfunctions of Sturm-Livoulli Boundary Value Problem, the vibrating string.

Unit IV. First Order Nonlinear Partial Differential Equation (15 Lectures)

First order quasi-linear PDE in two variables: Integral surfaces, Characteristic curves, Cauchy's method of characteristics for solving First order quasi-linear PDE in two variables.

First order nonlinear PDE in two variables , Characteristic equations , Characteristic strip, Cauchy problem and its solution for first order non linear PDE in two variables.

(Note : ODE stands for Ordinary Differential Equations and PDE stands for Partial Differential Equations)

Reference Books:

For Units I and II: 1. E.A. Codington, N. Levinson: Theory of ordinary differential Equations, Tata McGraw-Hill, India.

For Unit III:

1. G.F. Simmons: Differential equations with applications and historical notes, McGraw-Hill international edition.

For Unit IV: 1. Fritz John: Partial Differential Equations, Springer.

Recommended Books

1. Hurewicz W.: Lectures on ordinary differential equations, M.I.T. Press.

PSMT205/ PAMT205: Elementary Probability Theory

Unit I. Probability basics (15 Lectures)

Modelling Random Experiments: Introduction to probability, probability space, events. Classical probability spaces: uniform probability measure, fields, finite fields, finitely additive probability, Inclusion-exclusion principle, σ-fields, σ-fields generated by a family of sets, σ-field of Borel sets, Limit superior and limit inferior for a sequence of events.

Unit II. Probability measure (15 Lectures)

Probability measure, Continuity of probabilities, First Borel-Cantelli lemma, Discussion of Lebesgue measure on $\boldsymbol{\sigma}$-field of Borel subsets of $\mathbb{R}$ assuming its existence, Discussion of Lebesgue integral for non-negative Borel functions assuming its construction. Discrete and absolutely continuous probability measures, conditional probability, total probability formula, Bayes formula, Independent events.

Unit III. Random variables (15 Lectures)

Random variables, simple random variables, discrete and absolutely continuous random variables, distribution of a random variable, distribution function of a random variable, Bernoulli, Binomial, Poisson and Normal distributions, joint distribution of two random variables, Independent random variables, Expectation and variance of random variables both discrete and absolutely continuous.

Unit IV. Limit Theorems (15 Lectures)

Conditional expectations and their properties, characteristic functions, examples, Higher moments examples, Chebyshev inequality, Weak law of large numbers, Convergence of random variables, Kolmogorov strong law of large numbers (statement only), Central limit theorem (statement only).

Reference Books:

1. M. Capinski, Tomasz Zastawniak: Probability Through Problems.

Recommended Books

1. J. F. Rosenthal: A First Look at Rigorous Probability Theory, World Scientific.

2. Kai Lai Chung, Farid AitSahlia: Elementary Probability Theory, Springer Verlag.

3. Robert Ash: Probability and Measure Theory.

The scheme of examination for the revised course in the subject of Mathematics at the M.A./M.Sc. Part I Programme (semesters I & II) will be as follows.

Scheme of Examination

In each semester, the performance of the learners shall be evaluated into two parts. The learners performance in each course shall be assessed by a mid- semester test of 30 marks, active class participation 05 marks and attendance 05 marks in the first part, by conducting the Semester End Examinations with 60 marks in the second part.

External Theory examination of 60 marks:

(i) Duration:- Examination shall be of Two and Half hours duration. (ii) Theory Question Paper Pattern:-

1. There shall be five questions each of 12 marks.
2. On each unit there will be one question and the fifth one will be based on entire syllabus.
3. All questions shall be compulsory with internal choice within each question.
4. Each question may be subdivided into sub-questions a, b, c, ·· and the allocation of marks depend on the weightage of the topic.
5. Each question will be of 18 marks when marks of all the subquestions are added (including the options) in that question.

Questions		Marks
Q1	Based on Unit I	12
Q2	Based on Unit II	12
Q3	Based on Unit III	12
Q4	Based on Unit IV	12
Q5	Based on Units I,II,III& IV	12
Total Marks		60