

AC\_07/07/2023

Item No. 6.14 (N)

As Per NEP 2020

University of Mumbai



Title of the Program

- A- P. G. Diploma in Mathematics —  
B- M. Sc. (Mathematics) (Two Years) } 2023-24  
C- M. Sc. (Mathematics) (One Year) - 2027-28

Syllabus for  
Semester: I & II

Ref: GR dated 16<sup>th</sup> May, 2023 for Credit  
Structure of PG

## **Preamble**

### **1. Introduction**

With reference to GR No. NEP – 2022/ Pr. Kr. 09/ Vishi -3 Shikana Higher and Technical Education, Government of Maharashtra dated 16<sup>th</sup> May 2023, University of Mumbai has adopted New Education Policy 2020 for the Post Graduate Departments. Accordingly the revised academic curricula and syllabi are being brought into force from the academic year 2023–24. Mathematics has been fundamental to the development of science and technology. In recent decades, the extent of applications of Mathematics to real world problems has been increased by leaps and bounds. Taking into consideration the rapid changes in science and technology and new approaches in different areas of mathematics and related subjects like Physics, Statistics and Computer Sciences, the University of Mumbai has prepared the syllabus of M. Sc. Mathematics. The present syllabi of M. Sc. is for Semester I and Semester II have been designed as per U.G.C. Model curriculum so that the students learn Mathematics needed for these branches, learn basic concepts of Mathematics and are exposed to rigorous methods gently and slowly.

### **2. Aims & Objectives**

1. Deliver students a sufficient knowledge of fundamental principles, methods and clear perception of innumerable power of mathematical ideas and tools and know how to use them by modeling, solving and interpreting.
2. Reflecting the broad nature of the subject and developing mathematical tools for continuing further study in various fields of sciences.
3. Enhancing student's overall development and to equip them with mathematical modeling abilities, problem solving skills, creative talent and power of communication necessary for various kinds of employment.
4. Student should get adequate exposure to global and local concerns that explore them many aspects of Mathematical Sciences.

### **3. Learning Outcomes**

1. Enabling students to develop positive attitude towards mathematics as an interesting and valuable subject.
2. Enhancing student's overall development and to equip them with mathematical

modeling, abilities, problem solving skills, creative talent and power of communication.

3. Acquire good knowledge and understanding in advanced areas of mathematics and physics.

#### 4. Any other Point (If any)

#### 5. Baskets of Electives

##### Semester I

1. 513016150611: Commutative Algebra
2. 513016150612: Graph Theory
3. 513016150613: Programming in Python
4. 513016150614: Linear and Non Linear Programming

##### Semester II

1. 513016251611: Algebraic Number Theory
2. 513016251612: Advanced Counting Techniques in Discrete Mathematics
3. 513016251613: Algebraic Topology
4. 513016251614: Numerical Analysis

##### Semester III

1. 513016355611: Advanced Partial Differential Equations
2. 513016355612: Coding Theory
3. 513016355613: Matrix Algebra
4. 513016355614: Integral Transforms

##### Semester IV

1. 513016456511: Lie Algebra
2. 513016456512: Representation Theory of Finite Groups
3. 513016456513: Special Functions
4. 513016456514: Calculus on Manifolds

#### 6. Credit Structure of the Program

##### M.Sc. Mathematics

Sr. No.	Course Code	Course	Mandatory/ Elective	Theory/Practical	Credits
<b>Semester-I (Level 6.0)</b>					
1	5130161501	Algebra I	Mandatory	2TH + 2 PR	4
2	5130161502	Real Analysis	Mandatory	2TH + 2 PR	4
3	5130161503	Ordinary Differential Equations	Mandatory	2TH + 2 PR	4

4	5130161504	Discrete Mathematics and Number Theory	Mandatory	2 PR	2
5	5130161505	Research Methodology	Mandatory	2TH + 2 PR	4
<b>Students have to choose any one from the following elective courses</b>					
6	513016150611	Commutative Algebra	Elective	2TH + 2 PR	4
7	513016150612	Graph Theory	Elective	2TH + 2 PR	4
8	513016150613	Programming in Python	Elective	2TH + 2 PR	4
9	513016150614	Linear & Non Linear Programming	Elective	2TH + 2 PR	4
Total Credits :					22

Sr. No.	Course Code	Course	Mandatory /Elective	Theory/Practical	Credits
<b>Semester-II (Level 6.0)</b>					
1	5130162511	Algebra II	Mandatory	2TH + 2 PR	4
2	5130162512	Topology	Mandatory	2TH + 2 PR	4
3	5130162513	Complex Analysis	Mandatory	2TH + 2 PR	4
4	5130162514	Partial Differential Equations	Mandatory	2 PR	2
5	5130162515	OJT (On Job Training)/ FP(Field Project)	Mandatory	2TH + 2 PR (At least four week training at industry or Field work Certificate)	4
<b>Students have to choose any one from the following elective courses</b>					
6	513016251611	Algebraic Number Theory	Elective	2TH + 2 PR	4
7	513016251612	Advanced Counting Techniques in Discrete Mathematics	Elective	2TH + 2 PR	4
8	513016251613	Algebraic Topology	Elective	2TH + 2 PR	4
9	513016251614	Numerical Analysis	Elective	2TH + 2 PR	4
Total Credits :					22
Cumulative Credits (SEM-I and SEM-II):					44
<p><b>Note: Exit will be permitted as per National Education Policy-2020 (NEP 2020) on successful completion of first year (with 44 credits) and Post Graduate Diploma in Mathematics will be awarded by the University.</b></p>					

Sr. No.	Course Code	Course	Mandatory/ Elective	Theory/Practical	Credits
<b>Semester-III (Level 6.5)</b>					
1	5130163551	Algebra III	Mandatory	2TH + 2 PR	4
2	5130163552	Differential Geometry	Mandatory	2TH + 2 PR	4
3	5130163553	Measure Theory and Integration	Mandatory	2TH + 2 PR	4
4	5130163554	Probability Theory and Statistics	Mandatory	2 PR	2
5	5130163555	Research Project	Mandatory	-	4
<b>Students have to choose any one from the following elective courses</b>					
6	513016355611	Advanced Partial Differential Equations	Elective	2TH + 2 PR	4
7	513016355612	Coding Theory	Elective	2TH + 2 PR	4
8	513016355613	Matrix Algebra	Elective	2TH + 2 PR	4
9	513016355614	Integral Transforms	Elective	2TH + 2 PR	4
				Total Credits :	22
Cumulative Credits (SEM-I, SEM-II and SEM-III) :					66

Sr. No.	Course Code	Course	Mandatory /Elective	Theory/Practical	Credits
<b>Semester-IV (Level 6.5)</b>					
1	5130164561	Algebra IV	Mandatory	2TH + 2 PR	4
2	5130164562	Fourier Analysis	Mandatory	2TH + 2 PR	4
3	5130164563	Functional Analysis	Mandatory	2TH + 2 PR	4
4	5130164564	Research Project	Mandatory	-	6
<b>Students have to choose any one from the following elective courses</b>					
6	513016456511	Lie Algebra	Elective	2TH + 2 PR	4
7	513016456512	Representation Theory of Finite Groups	Elective	2TH + 2 PR	4
8	513016456513	Special Functions	Elective	2TH + 2 PR	4
9	513016456514	Calculus on Manifolds	Elective	2TH + 2 PR	4
				Total Credits :	22
Cumulative Credits (SEM-I, SEM-II, SEM-III and SEM-IV) :					88
<b>Note: As per NEP 2020 with successful completion of 2 years P.G. degree in M.Sc. Mathematics (with cumulative credits 88) will be awarded by the University.</b>					

## Letter Grades and Grade Points:

Semester GPA/ Program CGPA Semester/Program	% of Marks	Alpha-Sign / Letter Grade Result
9.00-10.00	90.0-100	O (Outstanding)
8.00 =< 9.00	80.0 =< 90.0	A+ (Excellent)
7.00 =< 8.00	70.0 =< 80.0	A (Very Good)
6.00 =< 7.00	60.0 =< 70.0	B+ (Good)
5.50 =< 6.00	55.0 =< 60.0	B (Above Average)
5.00 =< 5.50	50.0 =< 55.0	C (Average)
4.00 =< 5.00	40.0 =< 50.0	P (Pass)
Below 4.00	Below 40	F (Fail)
Ab (Absent)	-	Absent

## Teaching Pattern

1. Two theory lectures and two Practicals per week for each four credit courses.
2. Two Practicals per week for each Two credit courses.
3. Practicals to be conducted as per list provided for each course. In addition there shall be tutorials, seminars as required for the content of the course.

## Scheme of Examination

### A. For Four Credit Course

1. 50 : 50 scheme for all PG courses i. e. 50 marks for continuous internal assessment and 50 marks for end semester examination.
2. Separate head of passing is required for internal and end semester examination.
3. **Mid semester examination** shall be of 50 (40+10) marks.

Items	Marks	Duration	Remark
Internal Test (Theory component)	30	40 marks examination of 2 hours duration	Based on Unit-I and II
Internal Test (Practical component)	10		
Journal Certification and Attendance	10		At the end of semester based on all practicals as per the list provided in

			syllabus
Total	50		

4. The end semester examination shall be of 50 marks and 2.5 hours duration (It consist of two sections. Section-I (Theory Component 30 marks), Section-II (Practical Component 20 marks) )

Question No	Unit	Theory/ Practical	Max. Marks	Total maximum marks including options for each question
<b>Section-I (Theory Component)</b>				
Q1	Unit I and II	Theory	10	15
Q2	Unit III and IV	Theory	10	15
Q3	Common	Theory	10	15
<b>Section-II (Practical Component)</b>				
Q4	Unit III	Practical	10	15
Q5	Unit IV	Practical	10	15
Total			50	

**B. For Two Credit Course:**

25 : 25 scheme for all PG courses i.e. 25 marks for continuous assessment and 25 marks for end semester examination. Separate head of passing is required for internal and end semester examination.

**Mid semester examination** shall be of 25 (15+10) marks.

Items	Marks	Duration	Remark
Internal Test (Practical component)	15	15 marks examination of 1 hour duration	Based on Unit-I
Journal Certification and Attendance	10		At the end of semester based on all practicals as per the list provided in syllabus
Total	25		

The end semester examination shall be of 25 marks and 1.5 hours duration

Question No	Unit	Practical	Max. Marks	Total marks including options
Q1	Unit I	Practical	12	18
Q2	Unit II	Practical	13	20
Total Marks			25	

**Scheme of evaluation R8435 for M. Sc. Mathematics:**

- A) 100% internal evaluation scheme for University Department of Mathematics. Both Mid and End semester examinations will be conducted by the Department and answer books will be shown to the students.
  
- B) For affiliated PG centers end semester examination will be conducted by the University.



# SYLLABUS SEMESTER-I

# M. Sc. (Mathematics) Semester I & II

## Semester-I

### 5130161501: ALGEBRA I

#### Course Objectives:

1. Students will be introduced to the notion of dual space and double dual.
2. Students will be equipped to comprehend the annihilator of a subspace and its application to counting the dimension of a finite dimensional vector space.
3. The course aims equip students to identify the basic properties of determinants and understand their applications to solving system of equations.
4. The instructor will discuss how students can recognize nilpotent operators. The objective is to introduce students to the important notion of invariant subspaces. Through problem solving the course will discuss applications of the above concepts too.
5. On completion, the course will enable students to compare if the given forms are bilinear forms. The course will prove the spectral theorem with examples of spectral resolution. Symmetric bilinear forms and Sylvester's law, which are useful in applications will be spelt out too.

#### Course Outcomes:

1. The students will be able to demonstrate how the isomorphism between the dual space and the double dual works.
2. They will be able to employ the techniques of the theorems to compute the dual basis with respect to a given one. Also they will be analyze properties of determinants and use them for computation of determinants.
3. Students will be able to differentiate between invariant and non-invariant subspaces.
4. Students will be able to examine if a given form is a bilinear form.
5. Students will be able to convert a given matrix into the correct canonical form so that the signature of the symmetric form can be determined using Sylvester's law.

## Unit I. Dual spaces (7 Lectures and 8 Practicals)

1. Linear functionals, dual spaces of a vector space, dual basis (for finite dimensional vector spaces), annihilator  $W^\circ$  in the dual space  $V^*$  of a subspace  $W$  of a vector space  $V$  and dimension formula, a  $k$ -dimensional subspace of an  $n$ -dimensional vector space is intersection of  $n - k$  many hyperspaces. Double dual  $V^{**}$  of a vector space  $V$  and canonical embedding of  $V$  into  $V^{**}$ .  $V^{**}$  is isomorphic to  $V$  when  $V$  is of finite dimension. (ref:[1] Hoffman K. and Kunze R.).
2. Transpose  $T^t$  of a linear transformation  $T$ . For finite dimensional vector spaces:  $\text{rank}(T^t) = \text{rank } T$ ,  $\text{range}(T^t)$  is the annihilator of kernel ( $T$ ), matrix representing  $T^t$ . (ref:[1] Hoffman K and Kunze R)

## Unit II. Determinants & Characteristic Polynomial (8 Lectures and 7 Practicals)

Rank of a matrix. Matrix of a linear transformation, change of basis, similar matrices. Determinants as alternating  $n$ -forms, existence and uniqueness, Laplace expansion of determinant, determinants of products and transposes, adjoint of a matrix, determinants and invertible linear transformations, determinant of a linear transformation. Solution of system of linear equations using Cramer's rule. Eigen values and Eigen vectors of a linear transformation, Annihilating polynomial, Characteristic polynomial, minimal polynomial, Cayley-Hamilton theorem. (Reference for Unit II: [1] Hoffman K and Kunze R, Linear Algebra).

## Unit III. Triangulation of matrices (7 Lectures and 8 Practicals)

Triangulable and diagonalizable linear operators, invariant subspaces and simple matrix representation (for finite dimension). (ref: [5] N.S. Gopalkrishnan & [3] Serge Lang) Nilpotent linear transformations on finite dimensional vector spaces, index of a Nilpotent linear transformation. Linear independence of  $\{u, Nu, \dots, N^{k-1}u\}$  where  $N$  is a nilpotent linear transformation of index  $k \geq 2$  of a vector space  $V$  and  $u \in V$  with  $Nu \neq 0$ . (Ref: [2] I.N.Herstein).

For a nilpotent linear transformation  $N$  of a finite dimensional vector space  $V$  and for any subspace  $W$  of  $V$  which is invariant under  $N$ , there exists a subspace  $V_1$  of  $V$  such that  $V = W \oplus V_1$ . (Ref:[2] I.N.Herstein). Computations related to  $3 \times 3$  matrices, their minimal polynomials and Jordan canonical forms.

## Unit IV. Bilinear forms (8 Lectures and 7 Practicals)

1. Adjoint of a linear operator on an inner product space, unitary operators, self adjoint operators, normal operators. (ref:[1] Hoffman K and Kunze R). Spectral theorem for a normal operator on a finite dimensional complex inner product space. (ref:[4] Michael Artin, Ch. 8). Spectral resolution (examples only). (ref:[1] Hoffman K and Kunze R, sec 9.5).

2. Bilinear form, rank of a bilinear form, non-degenerate bilinear form and equivalent statements. (ref:[1] Hoffman K and Kunze R).
3. Symmetric bilinear forms, orthogonal basis and Sylvester's Law, signature of a Symmetric bilinear form. (ref:[4] Michael Artin).

## List of Practicals

1. Computation of Dual basis, annihilators and transpose.
2. Computations of determinants using theoretical properties.
3. Eigenvalues, eigenvectors, minimal and characteristic polynomial.
4. Computations on diagonalizable and triangulable linear operators.
5. Invariant subspaces, nilpotent transformations and examples on Jordan canonical forms.
6. Inner product spaces, Gram-Schmidt process, computing adjoint of a linear transformation.
7. Test the properties of Normal and unitary operator. Verify Spectral theorem.
8. Bilinear forms, non-degenerate forms and applications of Sylvester's law to find the signature of symmetric bilinear form.

## Recommended Text Books

1. Hoffman K and Kunze R: Linear Algebra, Prentice-Hall India.
2. I.N.Herstein: Topics in Algebra, Wiley-India.
3. Serge Lang: Linear Algebra, Springer-Verlag Undergraduate Text in Mathematics.
4. Michael Artin: Algebra, Prentice-Hall India.
5. N.S. Gopalkrishnan: University Algebra, New Age International, third edition, 2015.
6. Morris W. Hirsch and Stephen Smale, Differential Equations, Dynamical Systems, Linear Algebra, Elsevier.

## 5130161502: REAL ANALYSIS

### Course Objectives:

1. Students will be introduced to Euclidean spaces and topology on them.
2. Students will learn about operator norms and their properties.
3. Students will interpret the concept of differentiability in an abstract manner.
4. The course will equip the students with the knowledge of basic theorems like the inverse function theorem, implicit function theorem and mean value theorem.
5. The course aims to demonstrate explicitly how Riemann integration works and the role of Fubini's theorem in calculations in a wide range of examples.

### Course Outcomes

1. Students will be able to judge if the given Euclidean spaces are compact or not.
2. Students will be able to select Euclidean spaces that are compact, connected from the given list of spaces.
3. They will be able to predict if the given function is continuous/uniformly continuous or not.
4. Students will be able to test if the hypothesis of the implicit function theorem and implicit function theorem hold. If yes, they will further be able to explain the arising consequences.
5. Students will be able to evaluate the Riemann integral and illustrate how Fubini's theorem works.

### Unit I. Euclidean space $\mathbb{R}^n$ (7 Lectures and 8 Practicals)

Euclidean space  $\mathbb{R}^n$ : inner product  $\langle x, y \rangle = \sum_{j=1}^n x_j y_j$  of  $x = (x_1, \dots, x_n), y = (y_1, \dots, y_n) \in \mathbb{R}^n$  and properties, norm  $\|x\| = \sqrt{\sum_{j=1}^n x_j^2}$  of  $x = (x_1, \dots, x_n) \in \mathbb{R}^n$ , Cauchy-Schwarz inequality, properties of the norm function  $\|x\|$  on  $\mathbb{R}^n$ . (Ref. W. Rudin or M. Spivak).

Standard topology on  $\mathbb{R}^n$ : open subsets of  $\mathbb{R}^n$ , closed subsets of  $\mathbb{R}^n$ , interior  $A^\circ$  and boundary  $\partial A$  of a subset  $A$  of  $\mathbb{R}^n$ . (ref. M. Spivak)

Operator norm  $\|T\|$  of a linear transformation  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  ( $\|T\| = \sup\{\|T(v)\| : v \in \mathbb{R}^n \& \|v\| \leq 1\}$ ) and its properties such as: For all linear maps  $S, T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  and  $R : \mathbb{R}^m \rightarrow \mathbb{R}^k$

1.  $\|S + T\| \leq \|S\| + \|T\|$ ,
2.  $\|R \circ S\| \leq \|R\| \|S\|$ , and

3.  $\|cT\| = |c|\|T\| (c \in \mathbb{R})$ .

(Ref. C. C. Pugh or A. Browder)

Compactness: Open cover of a subset of  $\mathbb{R}^n$ , Compact subsets of  $\mathbb{R}^n$  (A subset  $K$  of  $\mathbb{R}^n$  is compact if every open cover of  $K$  contains a finite subcover), Heine-Borel theorem, Cantor Intersection Theorem. Bolzano-Weierstrass theorem: Any bounded sequence in  $\mathbb{R}^n$  has a convergent subsequence.

Continuity and uniform continuity.

### Unit II. Differentiable functions (8 Lectures and 7 Practicals)

Differentiable functions on  $\mathbb{R}^n$ , the total derivative  $(Df)_p$  of a differentiable function  $f : U \rightarrow \mathbb{R}^m$  at  $p \in U$  where  $U$  is open in  $\mathbb{R}^n$ , uniqueness of total derivative. Differentiability implies continuity.(ref:[1] C.C.Pugh or[2] A.Browder)

Chain rule. Applications of chain rule such as:

1. Let  $\gamma$  be a differentiable curve in an open subset  $U$  of  $\mathbb{R}^n$ . Let  $f : U \rightarrow \mathbb{R}$  be a differentiable function and let  $g(t) = f(\gamma(t))$ . Then  $g'(t) = \langle (\nabla f)(\gamma(t)), \gamma'(t) \rangle$ .
2. Computation of total derivatives of real valued functions such as
  - (a) the determinant function  $\det(X)$ ,  $X \in M_n(\mathbb{R})$ ,
  - (b) the Euclidean inner product function  $\langle x, y \rangle$ ,  $(x, y) \in \mathbb{R}^n \times \mathbb{R}^n$ .

(ref. M. Spivak, W. Rudin )

Results on total derivative: partial derivatives, directional derivative  $(D_u f)(p)$  of a function  $f$  at  $p$  in the direction of the unit vector, Jacobian matrix, Jacobian determinant. Results such as :

1. If the total derivative of a map  $f = (f_1, \dots, f_m) : U \rightarrow \mathbb{R}^m$  ( $U$  open subset of  $\mathbb{R}^n$ ) exists at  $p \in U$ , then all the partial derivatives  $\frac{\partial f_i}{\partial x_j}$  exists at  $p$ .
2. If all the partial derivatives  $\frac{\partial f_i}{\partial x_j}$  of a map  $f = (f_1, \dots, f_m) : U \rightarrow \mathbb{R}^m$  ( $U$  open subset of  $\mathbb{R}^n$ ) exist and are continuous on  $U$ , then  $f$  is differentiable.(ref. W. Rudin)

Derivatives of higher order,  $C^k$ -functions,  $C^\infty$ -functions.(ref. T. Apostol)

### Unit III. Inverse function theorem and Implicit function theorem (8 Lectures and 7 Practicals)

Theorem (Mean Value Inequality): Suppose  $f : U \rightarrow \mathbb{R}^m$  is differentiable on an open subset  $U$  of  $\mathbb{R}^n$  and there is a real number such that  $\|f(Df)_x\| \leq M \forall x \in U$ . If the segment

$[p, q]$  is contained in  $U$ , then  $\|f(q) - f(p)\| \leq M\|q - p\|$ . (ref. C. C. Pugh or A. Browder).

Mean Value Theorem: Let  $f : U \rightarrow \mathbb{R}^m$  is a differentiable on an open subset  $U$  of  $\mathbb{R}^n$ . Let  $p, q \in U$  such that the segment  $[p, q]$  is contained in  $U$ . Then for every vector  $\mathbf{v} \in \mathbb{R}^n$  there is a point  $x \in [p, q]$  such that  $\langle \mathbf{v}, f(q) - f(p) \rangle = \langle \mathbf{v}, (Df)_x(q - p) \rangle$ . (ref:T. Apostol) If  $f : U \rightarrow \mathbb{R}^m$  is differentiable on a connected open subset  $U$  of  $\mathbb{R}^n$  and  $(Df)_x = 0$  for all  $x \in U$ , then  $f$  is a constant map.

Contraction mapping theorem. Inverse function theorem, Implicit function theorem.(ref. A. Browder)

Taylor expansion for a real valued  $C^m$ -function defined on an open subset of  $\mathbb{R}^n$ , stationary points(critical points), maxima, minima, saddle points, second derivative test for extrema at a stationary point of a real valued  $C^2$ -function defined on an open subset of  $\mathbb{R}^n$ . Lagrange's method of undetermined multipliers. (ref. T. Apostol)

#### **Unit IV. Riemann Integration(8 Lectures and 7 Practicals)**

Riemann Integration over a rectangle in  $\mathbb{R}^n$ , Riemann Integrable functions, Continuous functions are Riemann integrable, Measure zero sets, Lebesgues Theorem(statement only), Fubini's Theorem and applications. (Reference for Unit IV: M. Spivak, Calculus on Manifolds).

#### **List of Practicals**

1. Examples on Euclidean spaces, operator norms and compactness.
2. Examples on continuity and uniform continuity of functions, compactness and connectedness.
3. Results on total derivatives.
4. Examples on Chain rule and partial derivatives;  $C^k, C^\infty$ -functions.
5. Examples on Mean value theorem, Inverse function theorem, implicit function theorem.
6. Examples on Maxima, minima, Taylor expansion and Lagrange's method of multipliers.
7. Examples on Riemann Integration, Measurable zero sets.
8. Examples on Riemann integrability and Fubini's theorem.

## Recommended Text Books

1. C. C. Pugh, Mathematical Analysis, Springer UTM.
2. A. Browder, Mathematical Analysis an Introduction, Springer.
3. T. Apostol, Mathematical Analysis, Narosa.
4. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill India.
5. M. Spivak, Calculus on Manifolds, Harper-Collins Publishers.

## 5130161503: ORDINARY DIFFERENTIAL EQUATIONS

### Course Outcomes

1. Through this course students are expected to understand the basic concepts of existence and uniqueness of solutions of Ordinary Differential Equations (ODEs).
2. In case of nonlinear ODEs, students will learn how to construct the sequence of approximate solutions converges to the exact solution if exact solution is not possible.
3. Students will be able to understand the qualitative features of solutions.
4. Students will be able to identify Sturm Liouville problems and to understand the special functions like Legendre's polynomials and Bessel's function.
5. Students will be to understand the applicability of the above concepts in different disciplins of Techonolgy.

### Unit I. Existence and Uniqueness of Solutions (7 Lectures and 8 Practicals)

Existence and Uniqueness of solutions to initial value problem of first order ODE- both autonomous, non autonomous,  $\epsilon$ -approximate solutions, Ascoli lemma, Cauchy-Peano existence theorem, Lipschitz condition, Picard's method of successive approximations, Picard-Lindelof theorem, System of Differential equations. Reduction of n-th order differential equations.

[Reference Unit-I of Theory of Ordinary Differential Equations; Earl A. Coddington and Norman Levinson, Tata McGraw Hill, India.]

### Unit II. Linear Equations with constant coefficients (7 Lectures and 8 Practicals)

The second order homogeneous equations, Initial value problem for second order equations, Uniqueness theorem, linear dependance and independence of solutions, Wronskian,a formula for the Wronskian, The second order non-homogeneous equations, The homogeneous equations of order  $n$ , Initial value problem for  $n^{th}$  order equations, The non-homogeneous equations of order  $n$ , Algebra of constant coefficient operators.

[Reference Unit-II of Earl A. Coddington, An Introduction to Ordinary Differential Equations, Prentice-Hall of India.]



### **Unit III. Linear Equations with variable coefficients (7 Lectures and 8 Practicals)**

Initial value problem for the homogeneous equation of order  $n$ , Existence and Uniqueness theorem, solution of the homogeneous equations, Wronskian and linear independence, reduction of the order of a homogeneous equation, the non-homogeneous equations of order  $n$ .

[Reference III of Earl A. Coddington, An Introduction to Ordinary Differential Equations, Prentice-Hall of India.]

### **Unit IV. Sturm-Liouville Problem & Qualitative Properties of Solutions (7 Lectures and 8 Practicals)**

Eigenvalue problem, Eigenvalues and Eigenfunctions, the vibrating string problem, Sturm Liouville problems, homogeneous and non-homogeneous boundary conditions, orthogonality property of eigenfunctions, Existence of Eigenvalues and Eigenfunctions, Sturm Separation theorem, Sturm comparison theorem. Power series solution of second order linear equations, ordinary points, singular points, regular singular points, existence of solution of homogeneous second order linear equation, solution of Legendre's equation, Legendre's polynomials, Rodrigues' formula, orthogonality conditions, Bessel differential equation, Bessel functions, Properties of Bessel function, orthogonality of Bessel functions.

[Reference Unit IV (24, 25), Unit V (Review), Unit-VII (40, 43, Appendix A) and Unit VIII (44, 45, 46, 47) : G. F. Simmons, Differential Equations with Applications and Historical Notes, Second Edition, Tata McGraw Hill, India]

### **Recommended Text Books**

1. Earl A., Coddington and Norman Levinson, Theory of Ordinary Differential Equations; Tata McGraw Hill, India.
2. Earl A. Coddington, An Introduction to Ordinary Differential Equations, Prentice-Hall of India.
3. G. F. Simmons, Differential Equations with Applications and Historical Notes, Second Edition, Tata McGraw Hill, India
4. Hurewicz W., Lectures on ordinary differential equations, M.I.T. Press.
5. Morris W. Hirsch and Stephen Smale, Differential Equations, Dynamical Systems, Linear Algebra, Elsevier.

## **PRACTICAL TOPICS ON ORDINARY DIFFERENTIAL EQUATIONS**

1. The Lipschitz condition.

2. Picard's theorem for solving initial value problems of the type  $\frac{dy}{dx} = f(x, y)$ ,  $y(x_0) = y_0$ .
3. Solving a system of first order differential equations of the form  $\frac{dx}{dt} = f(x, y)$ ,  $\frac{dy}{dt} = g(x, y)$ .
4. Linear dependence and independence of functions, Wronskian of two or more functions.
5. Solving an  $n$ th order homogeneous/non-homogeneous equation with constant coefficients. The particular integral may be evaluated using operator method or by the method of variation of parameters.
6. Solving linear, homogeneous differential equations of order  $n$  with variable coefficients, solving initial value problems. Reduction of the order of a homogeneous differential equation and solving the same. Solution of the corresponding non-homogeneous differential equation.
7. Solving eigenvalue problem, eigenvalues and eigenfunctions, solving vibrating string problem, Sturm Liouville problem, orthogonality of its eigenfunctions.
8. Sturm Separation theorem, Sturm Comparison theorem, Power series solution of second order linear differential equations, ordinary points, singular points, regular singularity.
9. Solution of Legendre's equation, legendre polynomials, Rodrigue's formula, orthogonality of legendre polynomials.
10. Bessel's differential equation, Bessel's functions and their properties, orthogonality of Bessel's functions.

**5130161504: DISCRETE MATHEMATICS AND NUMBER THEORY \*(2 credit course)**

**Course Objectives:**

1. The course will discuss linear diophantine equations and conditions for solving them.
2. The course aims to introduce notion of arithmetic functions and in particular the multiplicativity of the functions  $\tau$ ,  $\sigma$  and  $\phi$ .
3. The course aims to equip students with the methods of solving recurrence relations.
4. The course will enable students to comprehend and apply Polya theory of counting for various enumeration problems including those in other sciences.

**Course Outcomes:**

1. The students will be able to demonstrate how linear diophantine equations can be solved.

2. They will be able to employ the techniques of counting for many real-life problems.
3. Students will be able to solve many real life problems using inclusion-exclusion principle.
4. Students will be able to apply their advanced knowledge of counting to solve coloring problems and enumeration problems in other scientific domains.

### **Unit I. Basics of Number theory and counting (7 Lectures and 8 Practicals)**

Divisibility, Linear Diophantine equations, Congruences and their properties, Arithmetic functions  $\sigma, \tau, \phi$  and their multiplicative property, Permutations (with and without repetition) and combinations, Equivalence to solving equations with non-negative integral solutions. Selections with Repetitions from multisets.

### **Unit II. Techniques of counting and Polya theory (8 Lectures, 7 Practicals)**

Pigeon-hole principle, Inclusion Exclusion principle and its applications like counting derangements.

Methods to solve recurrence relations with constant coefficients: homogeneous and non-homogeneous recurrence relations.

Group action (specially for permutation groups), Orbit stabiliser theorem, Burnside Lemma and its applications, Cycle index, Polya's Formula, Applications of Polya's Formula to counting problems in sciences.

## **Recommended Text Books**

1. D. M. Burton, Introduction to Number Theory, McGraw-Hill.
2. Nadkarni and Telang, Introduction to Number Theory
3. V. Krishnamurthy: Combinatorics: Theory and applications, Affiliated East-West Press.
4. Richard A. Brualdi: Introductory Combinatorics, Pearson.
5. A. Tucker: Applied Combinatorics, John Wiley & Sons.
6. Norman L. Biggs: Discrete Mathematics, Oxford University Press.
7. Kenneth Rosen: Discrete Mathematics and its applications, Tata McGraw Hills.
8. Sharad S. Sane, Combinatorial Techniques, Hindustan Book Agency, 2013.

## List of Practicals

1. Arithmetic functions and congruences.
2. Distribution of distinguishable and indistinguishable objects into distinguishable and indistinguishable boxes (with conditions on distribution), Counting problems from multisets.
3. Inclusion-exclusion principle and recurrence relations.
4. Counting via Burnside's lemma and Polya's formula.

### 5130161505: RESEARCH METHODOLOGY

#### Course Objectives:

1. To understanding meaning of research Methodology
2. To learn how to define research problems
3. To learn how to review literature and research report
4. To learn how to use various tools available in Latex
5. To learn use of Mathematical software.

**Course Outcomes:** After completion of the course, learners would be able to:

1. Initiate some research work, research project of their own
2. Write Literature review and Research report.
3. Make Question paper, write Thesis, Resume in Latex
4. Draw pictures and Plot graphs in Latex, Make beamer presentation.
5. Use mathematical software for better understanding of Mathematical Concepts.

#### Unit I. Introduction to Research Methodology (7 Lectures, 8 Practicals)

Meaning of Research, Objectives of Research, Motivations in Research, Ethics of research (Legal issues, copyright, plagiarism), Types of Research, Research Approaches, Significance of Research, Research Methods v/s Methodology, Research and Scientific Methods, Research Process, Criteria of good Research.

Defining the Research Problem: What is Research Problem? Selecting the problem, Necessity of and Techniques in defining the problem.

## **Unit II. Research literature and review (7 Lectures, 8 Practicals)**

Review of Literature: Purpose of the Review, Identification of the related Literature, Organizing the related Literature, Archive.

Research Report: General format of the Research report, writing technical research report, Writing a research proposal, research paper, chapter writing, Ph.D. thesis, Erratum, Proof reading, Keywords and Phrases, Mathematical subject Classifications and indexing, Short communication, fast track communication of a research paper, Poster/Oral presentation, Plenary talks, Invited talks of a conference/ workshop.

## **Unit III. LaTeX (7 Lectures, 8 Practicals)**

Getting Started with LaTeX: Installing and using LaTeX, working with LaTeX online using Overleaf, Preamble, Basic Syntax: Creating a Title Page, Page Numbering and Headings, Modifying Text, Use packages. Fonts, symbols, indenting, paragraphs, line spacing, word spacing, titles and subtitles, Document class, page style, parts of the documents, table of contents, Command names and arguments, environments, declarations, Theorem like declarations, comments within text, Writing equations, Matrix, Tables, Math in Latex, Advanced Math in Latex. Page Layout: Titles, Abstract, Chapters, Sections, Equation references, citation. Table of contents, generating new commands, Figure handling, numbering, List of figures, List of tables, Generating bibliography and index. Beamer presentation, Tikz: drawing simple pictures, Function plotting, drawing pictures with nodes.

## **Unit IV. Mathematics Software (7 Lectures, 8 Practicals)**

Use of Mathematical software Geogebra / Scilab/ wxMaxima/ Mathematica/ Matlab.

## **List of Practicals**

1. Basic of Latex.
2. Practical on mathematical tools.
3. Practical on Table of contents, generating new commands.

## **Recommended Text Books:**

1. C.R. Kothari and Gaurav Garg, Research Methodology Methods and Techniques, New Age International Publishers, 2019.
2. Stefan Kottwitz, LaTeX Beginner's Guide, Packt Publications.

## **513016150611: COMMUTATIVE ALGEBRA**

### **Course Objectives:**

1. The course will discuss basics of the theory of ideals and Nakayama's lemma and its applications.
2. The course aims to introduce the notion of noetherian and artinian rings and primary decomposition in such rings.
3. The course aims to equip students with the knowledge of integral extensions and computations in explicit examples.
4. The course will enable students to comprehend the concept of Dedekind domains and factorization therein.

**Course Outcomes:**

1. The students will be able to demonstrate how the theory of ideals works in various types of rings.
2. They will be able to employ the techniques of localization to construct new rings and study properties therein.
3. Students will be able to learn about integral extensions and associated results.
4. Students will be able to apply their advanced knowledge of the above topics to the study of special kinds of rings: Dedekind domains.

**Unit I. Basics of rings and modules (7 Lectures, 8 Practicals)**

Basic operations with commutative rings and modules, Polynomial and power series rings, Prime and maximal ideals, Extension and contractions, Nil and Jacobson radicals, Nakayama's lemma.

**Unit II. Primary decomposition (8 Lectures, 7 Practicals)**

Local rings, Localization, Chain conditions: Noetherian and Artinian rings, Hilbert basis theorem, Associated primes, Primary decomposition and primary decomposition under localization.

**Unit III. Integral Extensions (7 Lectures, 8 Practicals)**

Integral extensions, Going up and going down theorems, The ring of integers in a quadratic extension of rationals, Noether normalization, Hilbert's nullstellensatz.

**Unit IV. Dedekind Domains (8 Lectures, 7 Practicals)**

Discrete valuation rings, Alternative characterizations of discrete valuation rings, Dedekind domains, Fractional ideals, Factorization of ideals in a Dedekind domain, Examples.

### **Recommended Text Books:**

1. S. Lang, Complex Analysis, Springer .
2. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley and Sons.
3. M. F. Atiyah and I. G. McDonald, Introduction to Commutative Algebra, Addison Wesley.

### **List of Practicals**

1. Computing prime and maximal ideals, nilradical and Jacobson radical in various rings.
2. Properties of extended and contracted ideals and applications of Nakayama's lemma.
3. Chain conditions and their properties.
4. Localizations and their properties, Examples of primary decompositions.
5. Integral extensions and their properties.
6. Calculating integral closures of various rings.
7. Discrete valuations and their properties.
8. Dedekind domains and their properties.

## 513016150612: GRAPH THEORY

**Course Outcomes:** Students should know the following:

1. Overview of Graph theory-Definition of basic concepts such as Graph, Subgraphs, Adjacency and incidence matrix, Eigen values of graph, Friendship Theorem. Degree, Connected graph, Components, Isomorphism, Bipartite graphs etc., Shortest path problem-Dijkstra's algorithm, Vertex and Edge connectivity-Result  $\kappa \leq \kappa' \leq \delta$ , Blocks, Block-cut point theorem, Construction of reliable communication network, Menger's theorem.
2. Cut vertices, Cut edges, Bond.
3. Trees, Characterizations of Trees, Spanning trees, Fundamental cycles.
4. Vector space associated with graph, Cayley's formula, Connector problem- Kruskal's algorithm, Proof of correctness, Binary and rooted trees, Huffman coding, Searching algorithms-BFS and DFS algorithms.
5. Eulerian Graphs- Characterization of Eulerian Graph, Randomly Eulerian graphs, Chinese postman problem- Fleury's algorithm with proof of correctness.
6. Hamiltonian graphs- Necessary condition, Dirac's theorem, Hamiltonian closure of a graph, Chvatal theorem, Degree majorisation, Maximum edges in a non-hamiltonian graph, Traveling salesman problem.
7. Matchings-augmenting path, Berge theorem, Matching in bipartite graph, Hall's theorem(Necessary and sufficient condition for complete Matching), Konig's theorem(Maximum matching is same as minimum vertex cover), Tutte's theorem, Personal assignment problem,
8. Independent sets and covering-  $\alpha + \beta = p$ , Gallai's theorem.
9. Ramsey theorem-Existence of  $r(k, l)$ , Upper bounds of  $r(k, l)$ , Lower bound for  $r(k, l) \geq 2m/2$  where  $m = \min\{k, l\}$ , Generalize Ramsey numbers- $r(k_1, k_2, \dots, k_n)$ , Graph Ramsey Theorem, Evaluation of  $r(G, H)$  when for simple graphs  $G = P_3, H = C_4$ .

**Unit I. Connectivity(7 Lectures and 8 Practicals)** Overview of Graph theory-Definition of basic concepts such as Graph, Subgraphs, Adjacency and incidence matrix, Eigen values of graph, Friendship Theorem, Degree, Connected graph, Components, Isomorphism, Bipartite graphs etc., Shortest path problem-Dijkstra's algorithm, Vertex and Edge connectivity-Result  $\kappa \leq \kappa' \leq \delta$ ; Blocks, Block-cut point theorem, Construction of reliable communication network, Menger's theorem.



**Unit II. Trees (7 Lectures and 8 Practicals)** Trees-Cut vertices, Cut edges, Bond, Characterizations of Trees, Spanning trees, Fundamental cycles, Vector space associated with graph, Cayley's formula, Connector problem- Kruskal's algorithm, Proof of correctness, Binary and rooted trees, Huffman coding, Searching algorithms-BFS and DFS algorithms.

**Unit III. Eulerian and Hamiltonian Graphs (7 Lectures and 8 Practicals)** Eulerian Graphs- Characterization of Eulerian Graph, Randomly Eulerian graphs, Chinese post- man problem- Fleury's algorithm with proof of correctness. Hamiltonian graphs- Necessary condition, Dirac's theorem, Hamiltonian closure of a graph, Chvatal theorem, Degree majorisation, Maximum edges in a non-hamiltonian graph, Traveling salesman problem.

**Unit IV. Matching and Ramsey Theory (7 Lectures and 8 Practicals)** Matchings- augmenting path, Berge theorem, Matching in bipartite graph, Halls theorem, Konig's theorem, Tutte's theorem, Personal assignment problem, Independent sets and covering-  $\alpha + \beta = p$ ; Gallai's theorem, Ramsey theorem-Existence of  $r(k; l)$ ; Upper and Lower bounds of  $r(k; l)$ .  $r(k; l) \geq 2^{m/2}$  where  $m = \min\{k, l\}$ ; Generalize Ramsey numbers-  $r(k_1, k_2, \dots, k_n)$ ; Graph Ramsey theorem, Evaluation of  $r(G; H)$  when for simple graphs  $G = P_3; H = C_4$ :

**Recommended Text Books:**

1. J.A. Bondy and U.S.R. Murty, Graph Theory with Applications, Elsevier.
2. J. A. Bondy and U.S. R. Murty, Graph Theory, GTM 244 Springer, 2008.
3. M. Behzad and A. Chartrand, Introduction to the Theory of Graphs, Allyn and Becon Inc., Boston, 1971.
4. K. Rosen, Discrete Mathematics and its Applications, Tata-McGraw Hill, 2011.
5. D.B.West, Introduction to Graph Theory, PHI, 2009.

## 513016150613:PROGRAMMING IN PYTHON

**Aim of this course:** Learn Basics of programming through Python Programming.

**Course Outcomes:** Students will learn:

1. Problem Solving strategies to solve problem with help of computer.
2. Variables, operators, data types and expressions in Python.
3. Iterations and Conditional statements, functions.

4. Create Strings & File, File Handling operations.
5. Data Structures, Handling data using SQL, MySQL.

## **Unit-I Problem solving strategies (7 Lectures and 8 Practicals)**

1. Problem Solving strategies: Problem analysis, formal definition of problem, Solution, top-down design, breaking a problem into sub problems, overview of the solution to the sub problems by writing step by step procedure (algorithm), flowcharts, pseudocodes
2. Python programming language:
  - (a) Variables, expressions and statements Values and types:int, float and str Variables: assignment statements, printing variable values, types of variables.
  - (b) Operators, operands and precedence: +, -, /, \*, \*\*, % PEMDAS(Rules of precedence)
  - (c) String operations: + : Concatenation, \* : Repetition
  - (d) Boolean, Comparison and Logical operators: Boolean operator: == Comparison operators: ==, !=, >, <, >=, <= Logical operators: and, or, not Mathematical functions: sin, cos, tan, log, sqrt etc. Keyboard input: raw input statement

## **Unit-II: Iterations and Conditional statements (7 Lectures and 8 Practicals)**

1. Conditional and alternative statements, Chained and Nested Conditionals: if, if-else, if-elif-else, nested if, nested if-else looping statements such as while, for etc Tables using while.
2. Functions: Calling functions: type, id Type conversion:int, float, str Type coercion Composition of functions User defined functions, Parameters and arguments

## **Unit-III Strings & File Handling (7 Lectures and 8 Practicals)**

1. Elementary Python Graphics

2. Strings and Lists in Python, Strings: Compound data types, Length(len function), String traversal: Using while statement, Using for statement, Comparison operators(>, <, ==), Lists and List operations, Use of range function, Accessing list elements, List membership and for loop, List operations, List updation: addition, removal or updation of elements of a list, Introduction to files and types of files (text, binary and CSV file): Relative and absolute paths, Text file: Opening, Modes (r, r+, w, w+, a, a+): Closing and opening a file with Clause, Writing/ Appending data using write() and writelines(): Reading text file using read(), readline() and readlines(), Seek and tell methods: Manipulation of data, Binary file: basic operations: Open using file open modes, Close a binary file: Import pickle module, dump() and load() method: Append and update operations, CSV file: import csv module, open/ close, write: Read from a csv file using csv.reader( ).

## **Unit-IV Data Structures, SQL, MySQL(7 Lectures and 8 Practicals)**

1. Data Structure:

Stack and operations : Implementation Of Stack Using List

Introduction To Queue: Operations On Queue

Implementation Of Queue Using List

2. SQL Commands

Database concepts: introduction and its need, Relational Data Model: relation, attribute, tuple, domain, degree, cardinality and keys, Structured Query Language: Introduction Data Definition Language and Data Manipulation Language, Data Type: Create, Use and Show and Drop database, Show, Create, Describe, Alter and Drop Table: Insert, Delete, Select, Operators, Aliasing, Distinct Clause, Where Clause : Meaning Of Null, Like, Update Command, Delete Command, Aggregate Functions, Joins: Cartesian Product On Two Tables: Equi-join And Natural Join.

3. Interface Python with mySQL Interface of python with an SQL database: Connecting SQL with Python, Performing Insert, Update, Delete Queries using Cursor: Display Data By Using fetchone(), fetchall(), Rowcount, Creating Database: Connectivity applications.

## **Recommended Text Books**

1. Downey, A. et al., How to think like a Computer Scientist: Learning with Python, John Wiley, 2015.
2. Goel, A., Computer Fundamentals, Pearson Education.

3. Lambert K. A., Fundamentals of Python - First Programs, Cengage Learning India, 2015.
4. Rajaraman, V., Computer Basics and C Programming, Prentice-Hall India.

## List of Practicals

1. Find factorial of non negative number  $n$
2. Find largest among a given set of numbers
3. Generating Fibonacci sequences.
4. Arrange  $n$  numbers in increasing order.
5. Type a Python code to accept an integer and check whether it's even , odd or prime.
6. Using Python evaluate Mathematical expressions similar to the following:
  - (a)  $\sqrt{2}$
  - (b)  $\sin(\pi/2)$
  - (c)  $\cos(\theta + \pi/2)$  where  $\theta$  is entered by the user.
  - (d)  $e^{(\log_{10}(x))}$  where  $x$  is entered by the user.
7. Circles, lines drawing.
8. Roots of quadratic equations.
9. Type a python code to display integers ranging from 1 to  $n$ . (value of  $n$  is entered by the user)
10. Type a Python code that displays integers 1 to 10 and their squares in the table format using while statement.
11. Type a Python code to display all odd numbers from 1 to 20 using lists and for statement.
12. Define a 33 matrix using lists in Python. Type a code to display each row of the matrix and each element of the matrix.
13. Type a Python code to define a function for swapping the two variable values using tuples.
14. Type a Python statement to create a file object test.dat.
15. Type a Python statement to put data "Hello World!" in the file test.dat.
16. Type a Python statement to close the file test.dat.

17. Type a Python statement to read the file test.dat.
18. Type a Python statement to read the file test.dat in a directory named test, which resides in share, which resides in user, which resides in the top-level directory of the system, called /.
19. Type a Python code to prompt the user for the name of a file and then try to open it. If the file doesn't exist, we don't want the program to crash.(Use try and except statements.)
20. Creating Records from Python Lists.
21. Creating re-usable functions to do all of this for us in the future.

## **513016150614:LINEAR AND NON-LINEAR PROGRAMMING**

**Course Outcomes:** After the completion of the course the student will be able to

1. Formulate real world problems into mathematical models and apply the theory of linear programming.
2. Understand the simplex method, Big-M method, Dual simplex method for linear programming.
3. Learn nonlinear programming with constraints and without constraints.

**Unit I. Linear Programming (7 Lectures and 8 Practicals)** Operations research and its scope, Necessity of operations research in industry, Linear programming problems, Convex sets, Simplex method, Theory of simplex method, Duality theory and sensitivity analysis, Dual simplex method.

**Unit II. Transportation Problems(7 Lectures and 8 Practicals)** Transportation and Assignment problems of linear programming, Sequencing theory and Traveling salesperson problem.

**Unit III. Unconstrained Optimization(7 Lectures and 8 Practicals)** First and second order conditions for local optima, One-Dimensional Search Methods: Golden Section Search, Fibonacci Search, Newtons Method, Secant Method, Gradient Methods, Steep-est Descent Methods.

**Unit IV. Constrained Optimization Problems(7 Lectures and 8 Practicals)** Problems with equality constraints, Tangent and normal spaces, Lagrange Multiplier Theorem, Second order conditions for equality constraints problems, Problems with inequality constraints, Karush-Kuhn-Tucker Theorem, Second order necessary conditions for inequality constraint problems.

**List of Practicals:**

1. The theoretical workings of the simplex method, Big-M method and Dual simplex method for making effective decision on variables so as to optimize the objective function.
2. To solve complex problems involved in various industries.
3. Demonstrate the optimized material distribution schedule using transportation model to minimize total distribution cost.
4. The theoretical workings of sequencing techniques for effective scheduling of jobs on machines.
5. The appropriate algorithm for allocation of resources to optimize the process of assignment
6. Understand and apply unconstrained optimization theory for continuous problems.
7. Single Variable Optimization Problems: Fibonacci Search Method, Golden Section Method, Newton's Method, Secant Method, steepest descent method.
8. Understand and apply constrained optimization theory for continuous problems.
9. Lagrange multipliers method necessary and sufficient conditions for optimality.
10. Problems with equality constraints, inequality constraints Kuhn-Tucker conditions.

**Recommended Text Books:**

1. H. A. Taha, Operations Research-An introduction, Macmillan Publishing Co. Inc., NY.
2. K. Swarup, P. K. Gupta and Man Mohan, Operations Research, S. Chand and sons, New Delhi.
3. S. S. Rao, Optimization Theory and Applications, Wiley Eastern Ltd, New Delhi.
4. G. Hadley, Linear Programming, Narosa Publishing House, 1995.24
5. F. S. Hillier and G.J. Lieberman, Introduction to Operations Research (Sixth Edition), McGraw Hill
6. Chong and Zak, Introduction to Optimization, Wiley-Interscience, 1996.
7. Rangarajan and K. Sundaram, A First Course in Optimization Theory, Cambridge University Press.

**SYLLABUS**  
**SEMESTER-II**

## Semester-II

### 5130162511: ALGEBRA II

#### Course Objectives:

1. To introduce students to various groups like dihedral groups, Matrix groups, Automorphism group, Inner automorphisms.
2. To make the students appreciate the structure theorem for finite abelian groups via examples.
3. To introduce the concepts of group actions and the orbit-stabilizer formula.
4. To prove classical theorems like the Sylow theorems and apply them for classification of various types of groups.
5. The course aims that students earn the knowledge of prime avoidance theorem, Chinese remainder theorem, and specialized rings like Euclidean domains, principal ideal domains, unique factorization domains, their inclusions and counter examples.

#### Course Outcomes:

1. Students will be able to compute subgroups, normal subgroups and centers in various groups.
2. The students will check and ascertain if a given group is a direct product of two subgroups.
3. After the course, the students will be able to compute orbits and stabilizers in many examples of group actions.
4. The students will learn applications of Sylow theorems like classification of various types of groups or checking simplicity of groups etc.
5. Students will be able to understand the Chinese remainder theorem and use it to solve a system of congruences.
6. Students will be able to test if the given rings are Euclidean domains, principal ideal domains or unique factorization domains.

#### Unit I. Groups and Group Homomorphisms (7 Lectures and 8 Practicals)

Review: Groups, subgroups, normal subgroups, center  $Z(G)$  of a group. The kernel of a homomorphism is a normal subgroup. Cyclic groups. Lagrange's theorem. The product set  $HK = \{hk \mid h \in H \ \& \ k \in K\}$  of two subgroups of a group  $G$ : Examples of groups such as Permutation groups, Dihedral groups, Matrix groups,  $U_n$ -the group of units of  $\mathbb{Z}_n$  (no questions be asked).

Quotient groups. First Isomorphism Theorem and the following two applications (reference: Algebra by Michael Artin)



1. Let  $\mathbb{C}^*$  be the multiplicative group of non-zero complex numbers and  $\mathbb{R} > 0$  be the multiplicative group of positive real numbers. Then the quotient group  $\mathbb{C}^*/U$  is isomorphic to  $\mathbb{R} > 0$ :
2. The quotient group  $GL_n(\mathbb{R})/SL_n(\mathbb{R})$  is isomorphic to the multiplicative group of non-zero real numbers  $\mathbb{R}^*$ :

Second and third isomorphism theorems for groups, applications.

Product of groups. The group  $\mathbb{Z}_m \times \mathbb{Z}_n$  is isomorphic to  $\mathbb{Z}_{mn}$  if and only if  $\gcd(m, n) = 1$ . Internal direct product (A group  $G$  is an internal direct product of two normal subgroups  $H, K$  if  $G = HK$  and every  $g \in G$  can be written as  $g = hk$  where  $h \in H; k \in K$  in a unique way). If  $H, K$  are two finite subgroups of a group, then  $|HK| = \frac{|H||K|}{|H \cap K|}$ . If  $H, K$  are two normal subgroups of a group  $G$  such that  $H \cap K = \{e\}$  and  $HK = G$ , then  $G$  is internal direct product of  $H$  and  $K$ . If a group  $G$  is an internal direct product of two normal subgroups  $H$  and  $K$  then  $G$  is isomorphic to  $H \times K$ . (Reference: Algebra by Michael Artin) Inner automorphisms, Automorphisms of a group. If  $G$  is a group, then  $A(G)$ ; the set of all automorphisms of  $G$ , is a group under composition. If  $G$  is a finite cyclic group of order  $r$ ; then  $A(G)$  is isomorphic to  $U_r$ ; the groups of all units of  $\mathbb{Z}_r$  under multiplication modulo  $r$ . For the infinite cyclic group  $Z$ ;  $A(Z)$  is isomorphic to  $\mathbb{Z}_2$ . Inner automorphisms of a group. (Reference: Topics in Algebra by I.N.Herstein). Structure theorem of Abelian groups (statement only) and applications (Reference: A first Course in Abstract Algebra by J. B. Fraleigh).

## Unit II. Groups acting on sets and Sylow theorems

Center of a group, centralizer or normalizer  $N(a)$  of an element  $a \in G$ ; conjugacy class  $C(a)$  of  $a$  in  $G$ : In finite group  $G$ ;  $|C(a)| = o(G)/o(N(a))$  and  $o(G) = \sum o(G)/o(N(a))$  where the summation is over one element in each conjugacy class, applications such as:

1. If  $G$  is a group of order  $p^n$  where  $p$  is a prime number, then  $Z(G) \neq \{e\}$ :
2. Any group of order  $p^2$ ; where  $p$  is a prime number, is Abelian. (Reference: Topics in Algebra by I. N. Herstein).

Groups acting on sets, Class equation, Cauchy's theorem: If  $p$  is a positive prime number and  $p|o(G)$  where  $G$  is finite group, then  $G$  has an element of order  $p$ . (Reference: Topics in Algebra by I. N. Herstein).

Simple groups,  $p$ -groups, Sylow theorems and applications (like testing simplicity of groups and)

1. There are exactly two isomorphism classes of groups of order 6:
2. Any group of order 15 is cyclic

(Reference for Sylow's theorems and applications: Algebra by Michael Artin).

### Unit III. Rings and Fields (7 Lectures and 8 Practicals)

Review: Rings (with unity), ideals, quotient rings, prime ideals, maximal ideals, ring homomorphisms, characteristic of a ring, first and second Isomorphism theorems for rings, correspondence theorem for rings (If  $f : R \rightarrow R'$  is a surjective ring homomorphism, then there is a 1 – 1 correspondence between the ideals of  $R$  containing the  $\ker f$  and the ideals of  $R'$ ). Integral domains, construction of the quotient field of an integral domain. (no questions be asked).

For a commutative ring  $R$  with unity:

1. An ideal  $M$  of  $R$  is a maximal ideal if and only if the quotient ring  $R/M$  is a field.
2. An ideal  $N$  of  $R$  is a prime ideal if and only if the quotient ring  $R/M$  is an integral domain.
3. Every maximal ideal is a prime ideal.
4. Every proper ideal is contained in a maximal ideal.
5. If an ideal  $I$  is contained in union of prime ideals  $P_1, P_2, \dots, P_n$ , then  $I$  is contained in some  $P_i$ .
6. If a prime ideal  $P$  contains an intersection of ideals  $I_1, I_2, \dots, I_n$ , then  $P$  contains some ideal  $I_j$ .

Rings of fractions, inverse and direct images of ideals, Comaximal ideals, Chinese Remainder Theorem in rings and its applications to congruences.

Definition of field, characteristic of a field, sub field of a field. A field contains a sub field isomorphic to  $\mathbb{Z}_p$  or  $\mathbb{Q}$ :

Polynomial ring  $F[X]$  over a field, irreducible polynomials over a field. Prime ideals, and maximal ideals of a Polynomial ring  $F[X]$  over a field  $F$ : A non-constant polynomial  $p(X)$  is irreducible in a polynomial ring  $F[X]$  over a field  $F$  if and only if the ideal  $(p(X))$  is a maximal ideal of  $F[X]$ : Unique Factorization Theorem for polynomials over a field (statement only).

### Unit IV. Divisibility in integral domains (7 Lectures and 8 Practicals)

Prime elements, irreducible elements, Unique Factorization Domains, Principle Ideal Domains, Gauss's lemma,  $\mathbb{Z}[X]$  is a UFD, irreducibility criterion, Eisenstein's criterion, Euclidean domains.  $\mathbb{Z}[\sqrt{-5}]$  is not a UFD.

Reference for Unit IV: Michael Artin: Algebra, Prentice-Hall India.

### Recommended Text Books

1. Michael Artin: Algebra, Prentice-Hall India.

2. I.N. Herstein: Topics in Algebra, Wiley-India.
3. J. B. Fraleigh, A first Course in Abstract Algebra, Narosa.
4. David Dummit, Richard Foot: Abstract Algebra, Wiley-India.
5. N. Jacobson, Basic Algebra I (2nd Ed.), Diver Publication.

## List of Practicals

1. Examples of groups, subgroups and applications of isomorphism theorems.
2. Quotient groups, products of groups.
3. Applications of structure theorem for abelian groups and existence of elements of certain order.
4. Group actions and computations of Sylow subgroups and elements of certain orders.
5. Applications of Sylow theorems.
6. Basic properties of different types of rings, quotient rings and ideals.
7. Prime elements, irreducible elements: their examples and counter-examples.
8. Eisenstein's criterion and details of special rings like Euclidean domains, Principal ideal domains and Unique factorization domains.

## 5130162512: TOPOLOGY

### Course Objective

1. To understand a formation of new spaces from old one using product, box and quotient topology.
2. To make student understand different spaces with different kinds of measuring relations (i.e. different kinds of open sets.)
3. Students will grasp the concepts of connectedness and compactness having different kinds of open sets.

### Course Outcomes

1. Students will learn different topological spaces on same set..
2. Student will understand identification process to achieve Quotient space.
3. They will perceive an idea of connectedness and compactness in all kinds of open sets.
4. Students will learn concept of metrization of a topological space.

### **Unit I. Topology and Topological spaces (8 Lectures, 7 Practicals)**

Topological spaces, basis, topology generated by basis, sub-basis, order topology, product topology, subspace topology, closed sets, limit points, closure, interior, continuous functions, homeomorphism, box topology, comparison of the box and product topologies,  $T_0$ ,  $T_1$  spaces, For a  $T_1$  space  $X$ ,  $x \in A \subset X$  is limit point of  $A$  if and only if every neighbourhood of  $x$  contains infinitely many points of  $A$ . Hausdorff space.

### **Unit II. Connected topological spaces (7 Lectures, 8 Practicals)**

Quotient spaces. Connected topological spaces, separation of a topological space, continuity and connected-ness, path-connected topological spaces, topologist's sine curve, the order square  $I_0^2$  is connected but not path connected. For  $\mathbb{R}$  equipped with usual topology, the infinite cartesian product  $\mathbb{R}^\omega$  in the product topology is connected but in box topology it is not. Connected components of a topological space, Path components of a topological space. Countability Axioms, first and second countable spaces, Separable spaces, Lindeloff spaces.

### **Unit III. Compact topological spaces (8 Lectures, 7 Practicals)**

Compact spaces, continuity and compactness, tube lemma, finite product of compact topological spaces is compact. Finite intersection property, the Lebesgue number lemma, uniform continuity theorem, compact Hausdorff space with no isolated points is uncountable. Limit point compact spaces, local compactness, one point compactification, Tychonoff theorem (statement only).

### **Unit IV. Metrizable spaces and Tychonoff theorem (7 Lectures, 8 Practicals)**

Metrizable spaces, Sequence lemma and metrizability criterion, separation axioms (regular and normal spaces). Every metrizable space is normal. A compact  $T_2$  space is a normal space. Urysohn lemma, Tietze extension theorem, Urysohn metrization theorem (Statement only).

### **Recommended Text Books**

1. J. F. Munkres: Topology, Pearson; 2 edition (January 7, 2000).
2. G. F. Simmons: Introduction to Topology and Modern Analysis, Tata McGraw Hill, 2004.

### **List of Practicals**

1. Generate basis from subbasis and topology from basis.
2. Determine limit points, closure and interior continuity of a function.
3. Using given identification form Quotient topology.

4. Check connectedness.
5. Check compactness.
6. Test features of one-point compactification.
7. Apply proof of Urysohn metrization theorem to specific problems.
8. Metrizable and separation axioms.

### **5130162513: COMPLEX ANALYSIS**

#### **Course Objectives**

1. Students will be introduced the difference of complex differentiation and real differentiation.
2. Students will be made to understand the smoothness of holomorphic maps.
3. The objective is to see how to simplify real integration through complex one.

#### **Course Outcomes**

1. In this course the students will learn about series of functions and power series. The concept of radius of convergence will be introduced and calculated.
2. This course gives insight of complex integration which is different from integration of real valued functions. In particular, Cauchy integral formula will be proved.
3. The students will learn that if a function is once (complex) differentiable then it is infinitely many times differentiable. This will be a sharp contrast with the theorems of real analysis.
4. Students will get exposure of finding of real or complex integration by using different kinds of singularities.
5. The various properties of Möbius transformations that have a wide variety of applications along with major theorems of theoretical interest like Cauchy-Goursat theorem, Morera's theorem, Rouché's theorem and Casorati-Weierstrass theorem will be studied.

#### **Unit I. Holomorphic functions (8 Lectures, 7 Practicals)**

Note: A complex differentiable function defined on an open subset of  $\mathbb{C}$  is called a holomorphic function.

Review: Complex numbers, Geometry of the complex plane, Weierstrass's M-test and its application to uniform convergence, Ratio and root test for convergence of series of complex numbers. (no questions to be asked).

Review: Stereographic projection, Sequence and series of complex numbers, Sequence and series of functions in  $\mathbb{C}$ , Complex differential functions, Chain rule for holomorphic function,

Power series of complex numbers, Radius of convergence of power series, Cauchy-Hadamard formula for radius of convergence of power series. Abel's theorem: let  $\sum_{n \geq 0} a_n(z - z_0)^n$  be a power series of radius of convergence  $R > 0$ . Then the function  $f(z)$  defined by  $f(z) = \sum a_n(z - z_0)^n$  is holomorphic on the open ball  $|z - z_0| < R$  and  $f'(z) = \sum_{n \geq 1} n a_n(z - z_0)^{n-1}$  for all  $|z - z_0| < R$ . Trigonometric functions, Applications of Abel's theorem to trigonometric functions.

Applications of the chain rule to define the logarithm as the inverse of exponential, branches of logarithm, principle branch  $l(z)$  of the logarithm and its derivative on  $\mathbb{C} \setminus \{z \in \mathbb{C} \mid \operatorname{Re}(z) \leq 0, \operatorname{Im}(z) = 0\}$ .

Introduction to Möbius transformations.

## **Unit II. Contour integration, Cauchy-Goursat theorem (7 Lectures, 8 Practicals)**

Contour integration, Cauchy-Goursat Theorem for a rectangular region or a triangular region. Cauchy's theorem (general domain), Cauchy integral formula, Cauchy's estimates, The index (winding number) of a closed curve, Primitives. Existence of primitives, Morera's theorem. Power series representation of holomorphic functions (Taylor's theorem).

## **Unit III. Properties of Holomorphic functions (7 Lectures, 8 Practicals)**

Entire functions, Liouville's theorem. Fundamental theorem of algebra. Zeros of holomorphic functions, Identity theorem. Counting zeros; Open Mapping Theorem, Maximum modulus theorem, Schwarz's lemma.

Isolated singularities: removable singularities and Removable singularity theorem, poles and essential singularities. Laurent Series development. Casorati-Weierstrass's theorem.

## **Unit IV. Residue calculus and Mobius transformation (8 Lectures, 7 Practicals)**

Residue Theorem and evaluation of standard types of integrals by the residue calculus method. Argument principle. Rouché's theorem,

### **Recommended Text Books**

1. J.B. Conway, Functions of one Complex variable, Springer.

2. A.R. Shastri: An introduction to complex analysis, Macmillan.
3. Serge Lang: Complex Analysis. Springer.
4. L.V. Ahlfors:Complex analysis, McGraw Hill.
5. R. Remmert: Theory of complex functions, Springer.
6. J.W. Brown and R.V. Churchill:Complex variables and Applications, McGraw-Hill.

### **List of Practicals**

1. Weierstrass M-test, root test, ratio test and computations of radius of convergence.
2. Abel's theorem and branch of the logarithm.
3. Cauchy Goursat theorem and applications of Cauchy's theorem.
4. Winding number, primitives and power series representation of functions.
5. Entire functions and zeroes of holomorphic functions.
6. Isolated singularities, removable singularities, essential singularities and poles.
7. Residue theorem and evaluation of integrals.
8. Möbius transformations.

### **5130162514: PARTIAL DIFFERENTIAL EQUATIONS**

#### **Course Outcomes**

1. Students are expected to understand the basic concepts and method of finding the solution of first and second order Partial Differential Equations (PDEs).
2. Students will be able to know the classification of second order PDEs, singularity and fundamental solution.
3. Students will be able to know the role of Green's function in the solution of Partial Differential Equations.

## **Unit-I: First Order Partial Differential Equations (7 Lectures and 8 Practicals)**

First order partial differential equations in two independent variables, Semilinear and Quasilinear equations in two independent variables, method of characteristics, the Characteristics Cauchy Problem, General solutions.

Non-linear equations in two independent variables: Monge Strip and Charpit Equations, Solution of Cauchy problem, Determination of Complete integral, solution of Cauchy problem,

[Reference Unit-I (1.1,1.2, 1.3, 1.4)of Phoolan Prasad & Renuka Ravindran, Partial Differential Equations, Wiley Eastern Limited, India]

## **Unit-II: Second Order Partial Differential Equations (7 Lectures and 8 Practicals)**

Classifications of second order partial differential equations in two and more than two independent variables, method of reduction to normal form, the Cauchy problem. Potential theory and elliptic differential equations, boundary value problems and Cauchy problem, Poisson's theorem, the mean value and the Maximum-Minimum properties

[Reference Unit-II (2.1, 2.2) of Phoolan Prasad & Renuka Ravindran, Partial Differential Equations, Wiley Eastern Limited, India]

Singularity functions and the fundamental solution, Green functions, Greens identities, Green's function for  $m$ -dimensions sphere of radius  $R$ , Green's functions Dirichlet problem in the plane, Neumann's function in the plane.

[Reference Unit-II (2.2.2):1. Phoolan Prasad & Renuka Ravindran, Partial Differential Equations, Wiley Eastern Limited, India and 2. Unit VIII of Yehuda Pinchover and Jacob Rubistein, An Introduction to Partial Differential Equations, Cambridge University Press]

### **List of Practicals**

1. Designing and analytical Solution of the Cauchy problem
2. Problems based on Monge Strip and Charpit Equations
3. Classification of the second order PDE by reducing into normal forms
4. Designing and the solution of the boundary value problems using Green's function.

### **Recommended Text Books**

1. Phoolan Prasad & Renuka Ravindran, Partial Differential Equations, Wiley Eastern Limited, India.
2. Yehuda Pinchover and Jacob Rubistein, An Introduction to Partial Differential Equations, Cambridge University Press.



3. T.Amaranath, An Elemetary Course in Partial Differential Equations, Narosa.
4. F. John, Partial Differential Equations, Springer publications.
5. G.B. Folland, Introduction to partial differential equations, Prentice Hall.

## **513016251611: ALGEBRAIC NUMBER THEORY**

### **Course Objectives:**

1. Students will be introduced to number fields and their properties.
2. Students will be trained to compute fundamental unit in given rings of integers of algebraic number fields.
3. The course aims equip students to apply quadratic reciprocity for computation of Legendre and Gauss symbols.
4. The instructor will discuss how factorization of ideals occurs and provide explicit criteria for the factorization in the case of quadratic number fields using Legendre symbols.
5. On completion, the course will enable students to compute certain class groups.

### **Course Outcomes:**

1. The students will be able to demonstrate the existence of fundamental unit and compute rings of integers of quadratic number fields.
2. They will be able to employ the techniques of quadratic reciprocity to calculate the given Legendre and Jacobi symbols.
3. Students will be able to identify the groups of units and factorize rational primes in quadratic number fields.
4. Students will be able to examine if the given ring of integers is a unique factorization domain or a principal ideal domain using the idea of class groups.

### **Unit I. Number Fields (8 Lectures, 7 Practicals)**

Field extensions, Number fields, Algebraic numbers, Integral extensions, Ring of integers in algebraic number fields, Fractional ideals, Prime factorization of ideals, Norm of an ideal, The group of units, Fundamental unit.

### **Unit II. Quadratic Reciprocity(7 Lectures, 8 Practicals)**

The Legendre symbol, Jacobi symbols, The laws of quadratic reciprocity with proof.

### **Unit III. Quadratic Fields: Factorization(8 Lectures, 7 Practicals)**

Quadratic fields, Real and imaginary quadratic fields, Ring of integers in a quadratic field, The group of units, Ideal Factorization in a quadratic field, Examples: The ring of Gaussian integers, The ring  $\mathbb{Z}[\sqrt{5}]$ , Factorization of rational primes in quadratic fields.

### **Unit IV. Imaginary Quadratic Fields: The Class Group (7 Lectures, 8 Practicals)**

The ideal class group of a quadratic field, Class groups of imaginary quadratic fields, The Minkowski lemma, Finiteness of the class group, Computation of class groups, Application to solving certain Diophantine equations.

#### **Recommended Text Books:**

1. P. Samuel, Algebraic Theory of Numbers, Dover, 1977.
2. M. Artin, Algebra, Prentice-Hall, India, 2000.
3. Marcus, Number Fields, Springer.
4. Algebraic Number Theory, T.I.F.R. Lecture Notes, 1966

#### **List of Practicals**

1. Algebraic extensions and their properties.
2. Computations of rings of integers and examples.
3. Quadratic reciprocity and computations of Legendre and Jacobi symbols.
4. Rings of integers of quadratic number fields and units therein.
5. Examples of ideal factorization in quadratic number fields.
6. Imaginary quadratic number fields and computations of class group.
7. Solving certain Diophantine equations.
8. Computations of class groups using Minkowski lemma.

### **513016251612: ADVANCED TECHNIQUES IN NUMBER THEORY AND DISCRETE MATHEMATICS**

#### **Course Objectives**

1. The students will be introduced to the classical Gauss's law of quadratic reciprocity. Along with this various applications of Möbius inversion will be studied along with applications.

2. Continued fractions will be introduced to the students. The important method of factorization, called the Pollard method will be introduced as a real-life application.
3. Students will be taught a topic of current research interest: designs. Basics of design theory will enable students to take up interesting projects.
4. The course aims to introduce the students to coding theory which is linear algebra over finite fields. This is also an area of interest of current research and students will be able to take up many further projects in the area.

### **Unit I. Quadratic Reciprocity and Further Combinatorics (7 Lectures and 8 Practicals)**

Cardano's and Ferrari's method, Quadratic congruences, Lifting congruences modulo higher prime powers, Gauss's quadratic reciprocity law, Möbius inversion, Applications of Möbius inversion, Permutations with Forbidden positions and Rook polynomials.

### **Unit II. Cryptography in real life (7 Lectures and 8 Practicals)**

Continued fractions, Properties of continued fractions and properties, Lucas theorem, Primality testing, Factorization using Pollard's  $\rho$ -method.

### **Unit III. Design Theory (7 Lectures and 8 Practicals)**

Incidence structures,  $t$ -designs, Hadamard matrices, Steiner systems and Fischer's inequality, Fano plane and Symmetric 2-designs.

### **Unit IV. Coding Theory (7 Lectures and 8 Practicals)**

Terminology of coding theory, Hamming bound, Singleton bound, Error-correction and detection, Linear codes, Cyclic codes, Weight enumerators and MacWilliam's theorem.

## **Recommended Text Books**

1. D. M. Burton, Introduction to Number Theory, McGraw-Hill.
2. S. Ling And C. Xing: Coding Theory: A first course.
3. J. H. van Lint, R. M. Wilson: A Course in Combinatorics, Cambridge.
4. Richard A. Brualdi: Introductory Combinatorics, Pearson.
5. A. Tucker: Applied Combinatorics, John Wiley & Sons.
6. Norman L. Biggs: Discrete Mathematics, Oxford University Press.
7. Kenneth Rosen: Discrete Mathematics and its applications, Tata McGraw Hills.
8. Sharad S. Sane, Combinatorial Techniques, Hindustan Book Agency, 2013.

## List of Practicals

1. Quadratic congruences and reciprocity law.
2. Möbius inversion and its applications.
3. Continued fractions and their properties.
4. Application to cryptography.
5. Examples of designs and classification of their types.
6. Fischer's inequality and symmetric designs.
7. Linear codes, cyclic codes, error correction and detection.
8. Bounds in various codes and weight enumerators.

### 513016251613: ALGEBRAIC TOPOLOGY

#### Course Objectives:

1. To study some features of surfaces and manifolds by using algebra.
2. To grasp covering of spaces through maps which helps to find universal covering.
3. To find chain of singular homology groups with some geometric properties through algebraic study.

#### Course Outcomes:

1. Students will be able to find Homotopy groups of some standard geometric objects.
2. Students will be able to understand nature of holes or handles of manifold using fundamental group.
3. Students can envisage nature of holes and handles of abstract object like Klein Bottle, Projective planes, etc.
4. In singular homology group, students will see the application of excision theorem.

#### Unit I. Fundamental Group (8 Lectures, 7 Practicals)

Homotopy. Path homotopy. The fundamental group. Simply connected spaces. Covering spaces. Path lifting and homotopy lifting lemma. Fundamental group of the circle.

#### Unit II. Fundamental group, Applications (8 Lectures, 7 Practicals)

Deformation retracts and Homotopy types. Fundamental group of  $S^n$ . Fundamental group of the projective space. Brouwer fixed point theorem. Fundamental theorem of algebra. Borsuk-Ulam theorem. Free Group, Free Product. Seifert-Van Kampen Theorem (without proof). Fundamental group of wedge of circles. Fundamental group of the torus.

### **Unit III. Universal Covering Spaces (8 Lectures, 7 Practicals)**

Equivalence of covering maps and Equivalences of covering spaces. The general lifting lemma. The necessary and sufficient condition of equivalence of two covering maps. Universal covering space. Conditional existence of Universal covering space. Example of a space with no universal covering space.

### **Unit IV. Singular Homology (8 Lectures, 7 Practicals)**

Singular  $p$ -simplex, singular chain complex, singular homology group, reduced singular homology group, acyclic space, zero dimensional singular homology group, induced homomorphism. Axioms for singular theory: Identity axiom, composition axiom, homotopy axiom, exactness axiom and commutativity axiom. Excision Theorem.

### **Recommended Text Books**

1. Alan Hatcher, Algebraic Topology, Cambridge University Press, 2002.
2. John Lee, Introduction to Topological Manifolds, Springer GTM, 2000.
3. James Munkres, Topology, Prentice Hall of India, 1992.
4. James Munkres, Elements of Algebraic Topology, Addison Wesley, 1984.2.

### **List of Practicals**

1. Examples of homotopy and path homotopy. Examples of covering maps
2. Determine fundamental group of circle.
3. Determine group homomorphism induced by continuous map.
4. Determine fundamental group of  $S^n$ ,  $\mathbb{R}P^2$ , cylinder, etc.
5. Application of Seifert-Van Kampen theorem to find fundamental group of figure eight, wedge of circles, torus, etc.
6. Examples of universal covering spaces for some topological objects.
7. Examples of singular  $p$ -simplex, chain complex and singular homology groups.
8. Examples of reduced singular homology group and relative homology group.
9. Example of relative homology group through excision theorem.

**513016251614: NUMERICAL ANALYSIS**

### **Course Outcomes:**

1. Students will be able to grasp the concept of numerical solution of various mathematical problems and corresponding errors.
2. Students will be able to understand the approximation of functions by least square method.
3. Students will be aware about applications of various numerical techniques in the solution of difference equations, ordinary and partial differential equations.

### **Unit I: Approximation of functions (7 Lectures and 8 Practicals)**

Least squares approximation, Weighted least squares method, Gram-Schmidt orthogonalizing process, Least squares approximation by Chebyshev polynomials. Discrete Fourier Transform and Fast Fourier Transform.

### **Unit II: Differential and Difference Equations (7 Lectures and 8 Practicals)**

Differential equations: Solutions of linear differential equations with constant coefficients. Euler's modified method, Runge-Kutta methods. Predictor corrector methods. Stability of numerical methods. Difference Equations: Linear difference equation with constant coefficients and methods of solving them.

### **Unit III: Numerical Integration (7 Lectures and 8 Practicals)**

Trapezoidal and Simpson's one third rule in composite forms, Gauss Legendre numerical integration, Gauss-Chebyshev numerical integration, Gauss-Hermite numerical integration, Gauss-Laguerre numerical integration with the derivation of all methods using the method of undetermined coefficients. Romberg's method. Gaussian quadratures, Multiple integrals.

### **Unit IV: Numerical solutions of Partial Differential Equations (7 Lectures and 8 Practicals)**

Classifications, Finite Difference approximations to derivatives, Finite difference methods of solving elliptic, parabolic and hyperbolic equations.

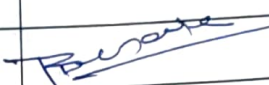
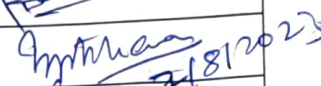
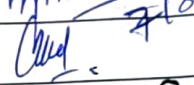
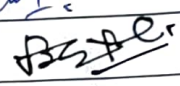
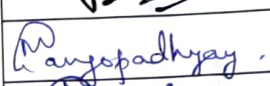
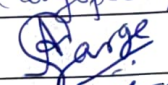
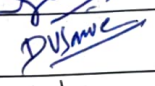


### **Recommended Text Books**

1. Jain, Iyengar, *Numerical Methods for Scientific and Engineering Problems*, New Age International.
2. S.S. Sastry, *Introductory Methods of Numerical Analysis*, Prentice-Hall, India.
3. K. E. Atkinson, *An Introduction to Numerical Analysis*, John Wiley and sons.
4. Erwin Kreyszig, *Advanced Engineering Mathematics*, Wiley John Wiley & Sons.
5. H. M. Antia, *Numerical Analysis* Hindustan Publications.

## PRACTICAL TOPICS ON NUMERICAL ANALYSIS

1. Approximation of functions by least squares approximation, weighted least squares method and Gram Schmidt orthogonalization.
2. Least squares approximation by Chebyshev polynomials, Discrete Fourier transform.
3. Solving linear differential equations with constant coefficients using Predictor-Corrector methods and Milne's method.
4. Solving two point linear boundary value problem using Galerkin's method, solving linear difference equations with constant coefficients.
5. Evaluating the integral of a function numerically using Trapezoidal rule, Simpson's 1/3rd rule, Simpson's 3/8th rule, Romberg's method. Multiple integration.
6. Numerical integration using Gaussian quadrature: Gauss Legendre integration, Gauss Chebyshev integration, Gauss Laguerre integration and Gauss Hermite integration.
7. Classification of partial differential equations and approximation of derivatives using finite differences methods.
8. Solving elliptic, parabolic and hyperbolic differential equations using finite difference schemes.

## Team for Creation of the Syllabus

Name	Department	Sign
Prof. B. S. Desale	University Department of Mathematics	
Prof. J. V. Prajapat		
Prof. Vinayak Kulkarni		
Dr. J. R. Junghare		
Dr. M. Gangopadhyay		
Dr. Anuradha Garge		
Mr. Deepak Sarwe		
Mr. Kamalakar Survade		
Mr. S. D. Shendage		



Sign of HOD

Prof. B. S. Desale,  
Head, Department of Mathematics

**Head**  
**Department of Mathematics**  
**University of Mumbai**



Sign of Dean

Name of the Dean: Prof. S. S. Garje  
Dean, Science and Technology



Appendix B

Justification for (M. Sc. Mathematics)

1.	Necessity for starting the course:	It is old course and now it is revised in accordance with NEP 2020
2.	Whether the UGC has recommended the course:	Yes. It is Government Aided course.
3.	Whether all the courses have commenced from the academic year 2023-24	Yes.
4.	The courses started by the University are self-financed, whether adequate number of eligible permanent faculties are available?:	No. It is government aided course. 50% of sanctioned posts are filled up.
5.	To give details regarding the duration of the Course and is it possible to compress the course?:	It is two year course and not possible to compress.
6.	The intake capacity of each course and no. of admissions given in the current academic year:	150; it is in process.
7.	Opportunities of Employability / Employment available after undertaking these courses:	Industrial Sector, Banking and Finance sector, Research and Teaching.

*B. S. Desale*

Sign of HOD

Prof. B. S. Desale,  
Head, Department of Mathematics

Head

Department of Mathematics  
University of Mumbai

*S. S. Garje*

Sign of Dean

Name of the Dean: Prof. S. S. Garje  
Dean, Science and Technology